Lab 3, Part 2: What does 'Random' motion look like? Describing Diffusion and Random Motion using Automatic Tracking

Introduction

Here is the broad structure of Lab 3, as it was presented to you last week:

- **Collect three videos;**
- **Analyze the first video to characterize the behavior of the random motion;**
- **Analyze all the videos to determine how the variation of parameters affects diffusion;** and
- **Write a lab report to synthesize the data gathered by you and the class and to summarize your findings regarding random motion and diffusion.**

Last week you began exploring the random motion of beads in fluids and began the process of characterizing this motion using histograms of different types of displacement at different times. This week you will continue to explore this random motion, finalizing your histograms (Δx, Δy, and r) at various times for video 1 (2 micron beads in water). When your histograms are finished and you have answered the questions asked in last week's lab document (repeated below), fully characterizing the nature of random motion, you should begin analyzing videos 2 and 3. In order to do this, it might help to have a little more information.

You will discover, as you build your total RMS displacement histograms, that the average displacement <u>magnitude</u>, i.e. the average distance traveled by the beads (let's call it $<r>$), deviates from zero: Every bead changes its position by some amount! As you remember, the distance traveled can be calculated as $r=\text{sqrt}(\Delta x^2+\Delta y^2)$, including only the squares of Δx and Δy—which are always positive and so add up to something bigger than zero. This equation indicates that instead of the distance traveled, r, we can measure the square of the distance traveled $r^2=\Delta x^2+\Delta y^2$, also called the Mean Squared Displacement or MSD. This makes the math a bit easier, and r^2 increases linearly with the measurement time interval, as you will see today.

The "diffusion constant," *D,* is defined to be the proportionality constant between the average displacement squared, r^2, of the diffusing object and the measurement time interval, Δt, over which the diffusion occurs. There is also a factor of 4 in there (for geometry reasons—a 4 in 2-dimensions, a 6 in 3-dimensions, a 2 in 1-dimension):

$$r^2=4D\Delta t$$

This Week in Lab

1. **Characterize the random motion of the beads in your first video:** Compare and contrast their motion to what you would expect for directed motion.
   a. Are the *average* total *x*- and *y*- displacements of the silica beads, $<\Delta x>$ and $<\Delta y>$, larger if you measure total displacements for larger time intervals? If the averages change, how do they change? If they don't change, why don't they change?
   b. Are the *individual* total *x*- and *y*- displacements of the silica beads, Δx and Δy, larger if you measure total displacements for larger time intervals? [*Hint: Make some histograms!*] If the

individual displacements change, how do they change? If they don't change, why don't they change? How are these individual displacements linked to the average displacements?

c. How does the root mean square (RMS) total displacement, $r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, change as a function of the measurement time interval? How will histograms of r change as the time interval increases? Does the average total displacement, <*r*> change? (Would it matter if we looked at the vector total displacement instead?)

2. **Examining Diffusion for Video 1:** How does the diffusion constant, D, depend on the measurement time interval? Using the information you have already created for your first video, investigate how the square of the average bead displacement, r^2, changes for different measurement time intervals, Δt. What is the diffusion constant for your video 1?

3. **Harvesting Data for Videos 2 and 3:** Now that you have fully analyzed video 1, you still have two videos to consider. These videos have been carefully chosen so that the class, as a whole, can make statements about how varying specific parameters affects the diffusion constant. You will not need to make histograms with the data that you collect, so you can look at fewer beads, but you do still need to keep track of and coordinate the frames from which you collect your data. Choose at least five suitable time intervals in the video (at least six frames) so beads move visibly, but not so far apart that it gets hard to distinguish nearby beads (examine at least 15 beads). Rather than tracking the beads manually, you may find **Automatic Tracking** helpful (see the technical skills document). Automatic tracking will collect the data for every frame—and you are welcome to use data from every frame—but you may also use only selected rows of the data produced by the automatic tracker.

4. **Analyzing Videos 2 and 3:** Analyzing this data will allow you to combine your results with other groups' work and make claims about how varying the investigated parameter affects the diffusion constant. As with video 1, you will need to create the MSD, r^2. Make a back-up of your data before you begin. Also, make a plan for how to manipulate your data BEFORE you do any calculations in your spreadsheet. (Planning now saves time later!) How does the square of the average bead displacement, r^2, change for different measurement time intervals, Δt, in each video? What are the diffusion constants for your videos 2 and 3?

Looking Ahead:

We did not explore the dependence of the diffusion constant, D, on temperature. Discuss with your group how you might expect D to vary as a function of temperature.

Using the data you have collected over the course of this lab, you will be expected to make an argument for a plausible expression for the diffusion constant, D, as a function of some (or all) of the following parameters: temperature, fluid viscosity, bead size, and bead mass. (An argument should contain a Claim, Data, and a Warrant—i.e., an explanation of how the claim is related to the data.)

Group # (Parameter) (each parameter will be investigated by two distinct groups, allowing you to 'double check' the results)	**Video 1** (condition for testing r^2 vs. t dependence)	**Video 2**	**Video 3**
1 & 2 (bead size)	2-micron silica beads in water	5-micron silica beads in water	1-micron silica beads in water
3 & 4 (fluid viscosity)	2-micron silica beads in water	2-micron silica beads in low viscosity glycerol/water mix	2-micron silica beads in high viscosity glycerol/water mix
5 & 6 (bead mass & viscosity)	2-micron polystyrene beads in water	2-micron polystyrene beads in low viscosity glycerol/water mix	2-micron polystyrene beads in high viscosity glycerol/water mix